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ITQs, Firm Dynamics and Wealth Distribution: Does Full Tradability Increase Inequality?

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Abstract Concerns over the re-distributive effects of individual transferable quotas (ITQ's) have led to restrictions on their tradability. We consider a general equilibrium model with firm dynamics to evaluate the redistributive impact of changing the tradability of ITQs. A change in tradability would happen, for example, if permits are allowed to be traded as a separate asset from ownership of an active firm. If the property right is associated with ownership of an active firm, the permit can be leased in each period but it is not possible to exit the industry and keep the right. However, allowing the permits to be traded as a separate asset has two effects. First, it leads to a greater concentration of production in the industry. Second, it directly converts a non-tradable asset into a tradable one, and this is equivalent to giving a lump sum transfer to all firms. The first effect implies a concentration in revenues, while the second implies a redistribution of wealth. We calibrate our model to match the observed increase in revenue inequality in the Northeast Multispecies (Groundfish) U.S. Fishery. We show

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that although observed revenue inequality—measured by the Gini coefficient—increases by 12 %, wealth inequality is reduced by 40 %.

Keywords ITQ · Wealth distribution · Firm dynamics · Inequality · Permit markets

1 Introduction

A crucial question in environmental and resource economics is why tradable output permits are not more widely adopted as a solution for environmental problems. The consensus appears to be that equity concerns are one important reason for the reluctance to use individual transferable quotas (ITQs) more widely.[1] In particular, the literature argues that the efficiency gains associated with tradable quotas will not be captured by small firms.[2]

These distributional concerns lead to restrictions in tradability of output permits, implying incompleteness of the right. For instance, in fisheries regulated with ITQs, if the property right is associated with ownership of an active eligible vessel the permit can be leased in each period but it is not possible to exit the fishery and keep the right. If the property right is assigned instead to the owner of the vessel and divorced from ownership of an active vessel, it could actually be traded as a separate asset and constitute a complete property right.

We show that a reform consisting of allowing the permits to be traded as a separate asset has two effects. On one hand, it leads to a greater concentration of production in the industry, as the most efficient firms will produce more. Second, it directly converts a non-tradable asset into a tradable one. This is equivalent to giving a lump sum transfer to all firms. The first effect implies a concentration in revenues, while the second implies a redistribution of wealth.

We consider a model of firm dynamics that builds on Weninger and Just (2002), Hopenhayn and Rogerson (1993) and Da-Rocha et al. (2014a). Firms are heterogeneous, but in contrast with the standard framework their distribution is not exogenous but rather determined endogenously by entry/exit decisions made by firms themselves.

We extend the model in Weninger and Just (2002)'s to a general equilibrium framework. The definition of a stationary equilibrium in a general equilibrium model with heterogeneous agents requires an invariant distribution of firms which is determined by agents' optimal policies, and also determines the agents' optimal choices. We use the Kolmogorov–Fokker–Planck equation to find that distribution.[3] We use the model to investigate the impact of changing the tradability of property rights on wealth distributions. The change in transferability affects entry/exit decisions, and also wealth distributions, which are endogenous in the economic environment.

[1] See Brandt (2005).

[2] In fisheries, for instance, there is an extensive literature on the relationship between tradability of ITQs and consolidation. See, for instance, Grafton et al. (2000), Fox et al. (2003), Kompas and Nu (2005) among others. There is also literature arguing that efficiency gains will be captured only by larger producers. See, for example Libecap (2007) or Olson (2011).

[3] The Kolmogorov–Fokker–Planck equation is widely used to describe population dynamics in ecology, biology, and finance, among other sciences. It has been used in economics by Merton (1975) in neoclassical growth models, by Dixit and Pindyck (1984) in a renewable resources model and by Da-Rocha and Pujolas (2011b) in fisheries. The use of Kolmogorov–Fokker–Planck equation to characterize the distribution of firms was suggested by Dixit and Pindyck (1994). There is a growing literature on general equilibrium models with heterogeneous firms that uses the Kolmogorov–Fokker–Planck equation to characterize the equilibrium invariant distribution of firms as in Luttmer (2007), Da-Rocha and Pujolas (2011a), Luttmer (2011), Luttmer (2012), Impullitti et al. (2013), Gourio and Roys (2014), Da-Rocha et al. (2014a, b, 2015), among others. Two good surveys are Gabaix (2009) and Luttmer (2010).

The Northeast Multispecies (Groundfish) U.S. fishery provides a good numerical example to illustrate the model. In 2010, a new management program implemented two significant changes: first, permits were allowed to participate in leasing and transfer programs without being activated by being placed onto a vessel and second, several constraints imposed on trade were removed.[4]

The 2010 final report on the performance of the Northeast Multispecies (Groundfish) Fishery reports the impact of the implementation of the New Management Plan. After remaining stable from 2007 to 2009, revenue inequality, measured by Gini coefficients, increased by 12 % in 2010.

We calibrate our model to match the observed increase in revenue inequality in that fishery after the reform and then use the model to compute the resulting unobserved change in wealth inequality. Our simulations show that when the property right is assigned to a person and divorced from the ownership of an active vessel, transferability of ITQs squeezes out small *vessels* but efficiency gains can also be captured through increases in the lease price for small *owners* by leasing quota, and wealth inequality is reduced by 40 %.

Our paper is also related to the literature on the distributional implications of alternative market-based control mechanisms. The "mechanism" that generates redistribution in our model is supported by the empirical findings of Brandt (2007). Furthermore, we are also able to show that wealth distribution among fishermen would actually improve with tradability. Because it computes how differences in the transferability of property rights affect market outcomes, the paper is also related to Gomez-Lobo et al. (2011), Grainger and Costello (2014) and Grainger and Costello (2015). A key difference between these papers and ours is that we compute the full wealth distribution, which is an endogenous object in our model.

The rest of the paper is organized as follows: Sect. 2 describes the economic environment. In Sect. 3 we characterize the equilibrium of the model and solve the closed form for the stationary distribution of firms' wealth. Section 4 calibrates the model with data from the US Northeast Multispecies Fishery, and finally Sect. 5 assesses the impacts of introducing free transferability into wealth distribution.

2 The Economic Environment

Assume a natural resource industry that is managed with tradable output permits q, where firms must own permits to exploit the resource legally. Total quota is determined exogenously, and for the sake of simplicity we normalize it to 1.

There are four markets in the economy: final goods, labor, an output permit lease market where trade takes place between incumbent firms, and a permit market where trade takes place between entrants and exiting firms. Taking output price as the numeraire, we denote by w, r_q and p_q, the labor, quota lease and quota ownership prices, respectively.[5]

[4] Those changes are reported in Measure 10 of the Magnuson–Stevens Fishery Conservation and Management Act Provisions; Fisheries of the Northeastern United States; Northeast (NE) Multispecies Fishery; Amendment 16; Final Rule.

[5] We do not consider many other types of distortion or other issues that could appear when regulating the environment with output permits. Examples of such distortions include imperfect enforcement as in Chavez and Salgado Cabrera (2005) and Hansen et al. (2014); international price externalities as in Burguet and Sempere (2010); transboundary resources as Garza-Gil (1998); distributional deadweight losses as in Thompson (2013); market power in intertemporal settings as in Armstrong (2008); and joint-ownership fishing exploitation as Escapa and Prellezo (2003).

We assume that all firms are ex-ante identical. However they are ex-post differentiated by a firm specific shock to production which is drawn from a distribution function $g(c)$.[6] We assume that the entry problem produces two decision rules: one for the optimal choice of the number of quotas, and, the other for the optimal entry decision. That is, firms choose how many quotas to hold and, at the same time, decide whether to enter the fishery. They decide to enter by assessing the expected value of operating a unit of capital (a vessel). Entry is costly and its cost is valued in terms of labor services, so if a firm decides to enter $c_e w$ has to be paid. If $W(c)$ denotes the value of operating a vessel when the shock is c and, c_*, the entry threshold, the expected value of entry can be expressed as: $W^e = \int_0^{c_*} W(c)g(c)dc - c_e w - p_q q$. After entry, entrants become incumbents.

Although we are interested in the stationary competitive equilibrium distribution of firms, note that individual firms change over time. Some of them expand production, hiring staff and borrowing quotas; others contract production, firing staff and leasing out quotas; and others exit the industry and sell their quotas. Therefore, the incumbent firms' decision problem produces two types of decision rule. On one hand, there are continuous decision rules for the optimal choice of output $y(c)$, labor $l(c)$ and the net demand for quotas (i.e. the number of quotas leased) $y(c) - q$, and, on the other hand, there is a discrete decision rule $d(c)$ for the optimal stay/exit decision.

We also assume that there is a fixed operating cost of c_f. If a firm wants to remain active then it must pay the fixed cost. The decision to exit depends on this period's employment $l(c)$, output $y(c)$, and permit leasing decisions $y(c) - q$. Conditional on this period's choices, $l(c)$, $y(c)$ and $y(c) - q$), the firm must assess the expected value of remaining in the industry, and must compare it to the present discounted value of profits associated with exiting the industry $p_q q$.

Given an initial guess for the exit threshold c_* made by rational expectations (a fixed point algorithm), potential entrants and incumbent firms can calculate the value of entry and, given market prices, solve their individual problems. Note that the distribution of firms for characteristic c depends on the support $[0, c_*)$. Therefore a (stationary) competitive equilibrium is a fixed point in a set of distributions $g(c) \in [0, c_*)$. This sequence of decisions by entrants and incumbent firms is explained graphically in Fig. 1.

3 Equilibrium

We consider a general equilibrium model with heterogeneous firms. First we solve the model when ITQs are permanent and fully tradable. This could be interpreted as the ITQ being dissociated from ownership of an active vessel, i.e. this is the case in which quotas can be leased for the current period but also permanently transferred without owning an active vessel. Later, we analyze the case in which ITQs can be leased for a given period but must to be associated with an active vessel.

3.1 The Problem of Incumbent Firms

Firms maximize profits subject to their available technology, $y = \sqrt{\frac{l}{c}}$. Note that we extend the model in Weninger and Just (2002) to a world in which quotas are a continuous variable

[6] This is a standard assumption in models with firm dynamics. See Hopenhayn (1992), Hopenhayn and Rogerson (1993) or Restuccia and Rogerson (2008).

$$
\begin{array}{llll}
 & & c > c_* \;\; exit & \\
Incumbent\,firms & \nearrow & & \\
g(c) \qquad , & \searrow & & \\
 & & c \leq c_* \;\; stay & \\
 & & & \searrow \\
 & & & \qquad New\ Incumbent\ firms \\
 & & & \nearrow \qquad\qquad g(c) \\
 & & enter\ with\ c \in (0, c_*] & \\
 & \nearrow & & \\
Potential\ entrants & & & \\
\end{array}
$$

$$W^e = \int_0^{c_*} W(c)g(c)dc - c_e w - p_q q$$

Fig. 1 Entrants and incumbents decision

and firms can lease part of their quota.[7] Also notice that when a firm chooses y, it is also implicitly choosing the net demand for quotas. Thus, intra-temporal profits are given by

$$\Pi = \max_{l,y} \;\; y - wl + r_q(q-y) - c_f,$$
$$s.t. \quad \begin{cases} y = \sqrt{\dfrac{l}{c}}, \\ q \geq y. \end{cases}$$

That is, profits are defined as output, y, minus labor costs, wl, plus net revenue from leasing quotas, $r_q(q-y)$, minus the fixed operating cost, c_f. From the f.o.c. we have, $l(w, r_q, c) = \left(\frac{1-r_q}{2w}\right)^2 c^{-1}$, $y(w, r_q, c) = \left(\frac{1-r_q}{2w}\right) c^{-1}$, and profits are given by

$$\Pi(w, r_q, c, q) = \pi(w, r_q)c^{-1} + r_q q - c_f.$$

where $\pi(w, r_q) = \frac{(1-r_q)^2}{4w}$.

Now the inter-temporal decision making of firms can be assessed. As in Weninger and Just (2002), we assume that the productivity shock c follows a geometric Brownian motion stochastic process with a positive expected growth rate, μ, i.e.

$$\frac{dc}{c} = \mu dt + \sigma dz,$$

where σ is the per-unit time volatility, and dz is the random increment to a Weiner process. Each firm has to weigh up its current and future potential profits against the benefits of selling its quota. Formally

$$W(c) = \max_{d \in \{stay, exit\}} \left\{ \pi(w, r_q)c^{-1} + (r_q q - c_f) + (1+\rho dt)^{-1} EW(c+dc), \quad p_q q \right\}$$
$$s.t. \;\; \frac{dc}{c} = \mu dt + \sigma dz,$$

[7] Our technology is in accordance with the fifty–fifty rule, i.e. 50 % of net revenues are accounted for by payments to crew members.

where $(1+\rho dt)^{-1}$ is the discount factor, $EW(c+dc)$ are the expected future profits, and $p_q q$ are the benefits of selling the permits and exiting the market. It is important to notice that in the competitive equilibrium all firms, regardless of their cost or productivity, sell their quotas at the same competitive price. That is, the price of quotas is independent of idiosyncratic characteristics. Finally, note that the value matching and smooth pasting conditions at the switching point c_* where firms choose to exit are $W(c_*) = p_q q$ and $W'(c_*) = 0$, respectively.[8]

The value function, $W(c)$, is obtained by solving the following ordinary second-order differential equation.

$$\rho W(c) = \pi(w, r_q)c^{-1} + (r_q q - c_f) + \mu W'(c) + \frac{\sigma^2}{2} W''(c)$$

with boundary conditions $W(c_*) = p_q q$ and $W'(c_*) = 0$. Proposition 1 characterizes the value function and the switching point.

Proposition 1 *The exit threshold, c_*, and the value function, $W(c)$, are given by*

$$c_* = \frac{(1+\beta)}{\beta} \frac{\rho}{(\rho+\mu-\sigma^2)} \left(\frac{\pi(w, r_q)}{\rho p_q q + c_f - r_q q} \right),$$

and

$$W(c) = \left(p_q q - \frac{(r_q q - c_f)}{\rho} \right) \frac{\beta}{1+\beta} \left(\frac{c}{c_*} \right)^{\beta} + \frac{\pi(w, r_q)c^{-1}}{\rho+\mu-\sigma^2} - \left(\frac{c_f - r_q q}{\rho} \right),$$

where $\beta = \left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right) + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2\rho}{\sigma^2}} > 1$ is the root of the standard quadratic equation associated with the geometric Brownian motion.

Proof See "Appendix 1". □

3.2 Invariant Distribution of Firms

For prices to be calculated the distribution of firms must be computed. In our economy, the distribution of firms is determined endogenously by entry/exit decisions made by firms themselves. To find that distribution we start by rewriting the model in logarithms, i.e. $x = \log(c/c_*)$, and apply the the Fokker–Planck–Kolmogorov equation

$$\frac{\partial M(x,t)}{\partial t} = -\hat{\mu} \frac{\partial M(x,t)}{\partial x} + \frac{\sigma^2}{2} \frac{\partial^2 M(x,t)}{\partial x^2} + \varepsilon M(x,t).$$

of the stochastic process $dx = \hat{\mu} dt + \sigma dz$, where $M(x,t)$ is the mass of firms over the variable x, $\varepsilon M(x,t)$ represents the new firms that enter with productivity x at time t, and $\hat{\mu} = \mu - \frac{\sigma^2}{2}$.[9] The partial differential equation is supplemented by the boundary condition $M(0,t) = 0$. This boundary condition guarantees that the mass of firms at the boundary, where firms decide to leave, is zero.

[8] See Dixit and Pindyck (1994) Chapter 4 for a formal definition and justification of these conditions.

[9] The Kolmogorov–Fokker–Planck equation is obtained by applying a simple Markov principle to the transition density function of the continuous stochastic process. Kolmogorov in the 1930's and Feller at the end of the 40's characterized the Kolmogorov–Fokker–Planck equation in such a way. For a formal characterization of the forward Kolmogorov equation and its relationship with the Markov stochastic process, see Mangel (2006).

We are interested in the steady state distribution with a constant mass of firms, $M(x,t) = Mf(x)$. Therefore the Fokker–Planck–Kolmogorov equation becomes

$$-\hat{\mu} f'(x) + \frac{\sigma^2}{2} f''(x) + \varepsilon f(x) = 0.$$

The stationary pdf is the solution of the boundary-value problem that consists of solving the following second order differential equation:

$$f''(x) - \gamma_1 f'(x) + \gamma_2 f(x) = 0,$$

where the constants γ_1 and γ_2 are given by $\gamma_1 = \frac{2\hat{\mu}}{\sigma^2} > 0$ and $\gamma_2 = \frac{2\varepsilon}{\sigma^2} > 0$, with the boundary condition $f(0) = 0$.

Proposition 2 *The solution of the stationary pdf satisfies* $\gamma_1^2 = 4\gamma_2$. *Therefore, the stationary entry rate* ε *is* $\dfrac{\hat{\mu}^2}{2\sigma^2}$.

Proof See "Appendix 2". □

We solve the boundary-value problem using Laplace transforms.[10] Applying Laplace transforms to the second order differential equation, we have

$$(s^2 - \gamma_1 s + \gamma_2)\mathscr{L}[f(x)] - (s - \gamma_1) f(0) - f'(0) = 0.$$

Using $\gamma_1^2 = 4\gamma_2$ and the boundary condition, we find:

$$\mathscr{L}[f(x)] = \frac{f'(0)}{(s-r)^2},$$

where $r = \dfrac{\hat{\mu}}{\sigma^2} > 0$. We obtain the solution by solving the Laplace inverses, $\mathscr{L}^{-1}$, given by

$$f(x) = \mathscr{L}^{-1}\left[\frac{f'(0)}{(s-r)^2}\right] = f'(0) x e^{rx}.$$

Finally $\int_{-\infty}^{0} f(x) = f'(0) \int_{-\infty}^{0} x e^{rx} = 1$ implies that $f(x) = -r^2 x e^{rx}$, given that $f'(0) = -r^2$.

Note that $f(x)$ is bounded and well defined. The distribution is stationary because the stochastic effect compensates the deterministic drift effect. Although c increases in expected terms given enough time, some of the firms are lucky and c decreases.

Moreover at the limit, firms do not leave the distribution. The exit rate at $x = -\infty$ is $-\mu f(-\infty) + \sigma^2/2 f'(-\infty)$.[11] At the lower bound, $-\infty$, the exit rate is zero. That is, $\underline{x} = -\infty$ is a natural reflecting barrier. The distribution is only loosing firms at $x = 0$, because firms find optimal to leave the industry. Therefore, an endogenous pdf can be found without imposing an exogenous reflecting barrier.[12] Finally, using $c = c_* e^x$, we recover the stationary cost distribution. Proposition 3 summarizes our findings.

[10] Laplace transforms are given by $\mathscr{L}[f'(x)] = s\mathscr{L}[f(x)] - f(0)$ and $\mathscr{L}[f''(x)] = s^2\mathscr{L}[f(x)] - sf(0) - f'(0)$.

[11] See "Appendix 2". Dixit and Pindyck (1994) offers an intuitive argument.

[12] With an exogenous reflecting barrier, the solution would be a bounded Pareto distribution. In fact, if it is assumed that $c \subset [\underline{c}, c_*]$ with an exogenous reflecting barrier at $\underline{c}$, the solution can be obtained by solving a two-boundary value problem. However this solution would depend on the (exogenous) reflecting barrier imposed.

Proposition 3 *The invariant distribution of firms is* $g(c) = -\frac{(1+\xi)^2}{c_*}\log(c/c_*)\left(\frac{c}{c_*}\right)^{\xi}$ *where the tail index is* $\xi = \frac{\mu}{\sigma^2} - \frac{3}{2}$.

Note the measures used in the report about the performance of the Northeast Multispecies fishery include the Lorenz curves of revenue.[13] It must be remarked that Lorenz curves are well defined for infinite support just as the equilibrium distribution of firms in our model is.[14]

3.3 Problem of Entrants

Given the value function $W(c)$, the gross value of entry W^e can be computed by using $g(c)$. That is

$$W^e = \int_0^{c_*} W(c)g(c)dc - wc_e - p_q q.$$

Potential entrants choose the number of quotas by solving

$$q^* \in \arg\max_q \int_0^{c_*} W(c)g(c)dc - wc_e - p_q q.$$

Notice that since entrants are ex-ante identical they all choose the same value for q. Entrants decide to enter if $W^e > 0$ for the chosen q. The result below provides a non arbitrage condition relating the price of permanently selling an ITQ to the leasing price. It implies equivalence between selling the permit and leasing it for an infinite number of periods.

Proposition 4 *In an equilibrium with exit, the no-arbitrage condition* $p_q = \frac{r_q}{\rho}$ *holds.*

Proof See "Appendix 3". □

Finally, notice that in an equilibrium with entry W^e must be zero, since otherwise additional firms would enter.

3.4 Feasibility Conditions

To close the model we need to define feasibility conditions. Feasibility in the model requires resource balance in the output market, the leasing quota market and the labor market. By normalizing the total quota at 1, it results that feasibility in the output market is given by $Mq = 1$. Feasibility in the leasing ITQ market implies that the aggregate excess demand function $M\left(\int_0^{c_*} y(c)g(c)dc - q\right)$ is equal to zero. That is

$$q = \int_0^{c_*} y(c)g(c)dc.$$

Finally, equilibrium in the labor market implies that

$$1 - \varepsilon M c_e = M\int_0^{c_*} l(c)g(c)dc,$$

where the total labor supply is normalized to 1, and $\varepsilon M c_e$, the entry cost multiplied by the mass of entrants, is the labor force allocated to produce the entry cost.

[13] See Kitts et al. (2011).

[14] Infinite support is standard in wealth studies. Heterogeneity in wages is assumed to be Log-normal distributed (with infinite support). Other empirical studies use statistics based on distributions with infinite supports. For example, Zwip's low (applied to cities, landscape, etc.) is based on GBM with infinite supports.

3.5 Definition of Equilibrium

A stationary equilibrium is an invariant cost distribution $g(c)$, a mass of firms M, a number of permits q, permit prices p_q and r_q, wage rate w, incumbents and entrants value functions $W(c)$, W^e, individual decision rules $l(c)$, $y(c)$, $\pi(c)$ and a threshold c_*, such that:

(1) (Firm optimization) Given prices (r_q, p_q, w), the *entry* functions, and $W(c)$ and W^e solve incumbent and entrant problems, $l(c)$, $y(c)$, $\pi(c)$ are optimal policy functions and and c_* is the threshold associated with the optimal exit rule.
(2) (Free-entry and optimal quota) Potential entrants choose quotas q and make zero profits, i.e. $W^e = 0$.
(3) (non-arbitrage condition) $p_q = \dfrac{r_q}{\rho}$.
(4) (Market clearing-feasibility) Given individual decision rules, prices (r_q, p_q, w) solve

$$\begin{aligned} 1 - \varepsilon M c_e &= M \int_0^{c_*} l(c) g(c) dc, \\ q &= \int_0^{c_*} y(c) g(c) dc, \\ Mq &= 1. \end{aligned}$$

(5) (Invariant distribution) $f(x)$, satisfies the Kolmogorov–Fokker–Planck equation.

Note that the definition of equilibrium is similar to the standard definition in Hopenhayn and Rogerson (1993) and Restuccia and Rogerson (2008). The main difference is that Hopenhayn and Rogerson (1993) and Restuccia and Rogerson (2008) consider a discrete time model. However, obvious equivalences appear. In fact, assuming Brownian motion is equivalent to assuming an AR(1) stochastic process, and the Kolmogorov–Fokker–Planck equation is the continuous time version of the (endogenous) discrete Markovian chain.

3.6 Equilibrium When Quotas are not Fully Tradable

In some fisheries distribution and other concerns have led to limits being placed on the trading of quotas. These limits on transferability can be implemented in many different ways. For example, Arnason (2002) reports that in most Canadian quota-managed fisheries ITQs are only transferable within the year, that is quotas can be leased only for a given period and therefore are not permanent and fully transferable.[15] In other fisheries, ITQs are distributed on an active vessel basis, and not directly to vessel owners. This means that only those individuals or firms that own eligible active vessels can hold quotas.

Consider the case where quotas must be associated with an active vessel.[16] In that case, one must own an eligible vessel to own a quota. As in the fully transferable case, there is an equilibrium at which some extremely unproductive vessels exit permanently, and some intermediate vessels fish the minimum amount required for them to be considered active, and lease the (remaining) quota if lease prices are high enough to justify paying the idling (or storage, or minimum activity) costs. That is, once one has an eligible vessel one can obtain revenues from leasing one's quota.

In the limited transferability case a permanent transfer of the quota should include the cost of the quota and the cost of the boat. That is, the price of the "package" is the expected value

[15] That is, from a legal standpoint an individual fishing quota is simply a fishing license with a certain tuple of stipulations.

[16] We are indebted to an anonymous referee for suggesting us this exposition.

of operating the boat, $W(c)$. The question then is what the relevant "exit option" is when the quota is tied to an eligible vessel. The "net" option of the exiting party is the benefit from leaving the market when productivity is so low that the revenues from leasing the quota do not justify paying the idling (or storage, or minimum activity) costs. That is, the equilibrium is the limit case of the full transferable case when $p_q \to 0$. Thus firms solve the following optimization problem

$$W(c) = \max_{d \in \{stay, exit\}} \left\{ \pi(w, r_q)c^{-1} + (r_q q - c_f) + (1 + \rho dt)^{-1} EW(c + dc), \ \ 0 \right\}$$

$$s.t. \ \frac{dc}{c} = \mu dt + \sigma dz.$$

in which the profits from operating a vessel are contrasted with the value of exiting the industry (which is normalized to zero).[17] There is a cost, c_*, for which owners find it optimal to exit the industry.

Corollary 1 *When $p_q = 0$, c_* and $W(c)$ are given by*

$$c_* = \frac{(1+\beta)}{\beta} \frac{\rho}{(\rho + \mu - \sigma^2)} \left(\frac{\pi(w, r_q)}{c_f - r_q q} \right),$$

and

$$W(c) = \frac{(c_f - r_q q)}{\rho} \frac{\beta}{1+\beta} \left(\frac{c}{c_*} \right)^{\beta} + \frac{\pi(w, r_q)c^{-1}}{\rho + \mu - \sigma^2} - \left(\frac{c_f - r_q q}{\rho} \right).$$

Summarizing, there is a key difference between the lessor of quotas in the limited transferability case and the lessor of quotas in the non limited transferability case. With permanent transferability, the landlord of the quota can rent the whole quota as it does not need to be included in the group of vessels that makes at least one trip per year. This is the type of landlord reported by Brandt (2007): they are quota owners who choose to cease harvesting. However, in the case of non permanent transferability the lessor of quotas must necessarily be included in the group of vessels that makes at least one trip per year, and therefore has to incur the cost of idling.

4 The Northeast Multispecies (Groundfish) Fishery

The New England fishery is located in portions of the Atlantic Ocean off the States of Maine, New Hampshire, Massachusetts, Rhode Island, and Connecticut (See the Magnuson–Stevens Fishery Conservation and Management Act, SEC. 302). Two primary regulatory bodies govern the conservation and management of this fishery: the New England Fishery Management Council and the North East Regional Office of the National Oceanic and Atmospheric Administration (NOAA).

On 1 May 2010, a new management program was implemented for the New England Groundfish Fishery.[18] The new Groundfish management program contained two significant changes.[19] The first allowed permits held in confirmation of permit history to participate in the leasing and transfer programs without being activated by being placed onto a vessel.[20]

[17] Those who cease activity cannot lease quota.

[18] Amendment 16 to the Northeast Multispecies Fishery Management Plan (FMP).

[19] Final ACT: Federal Register / Vol. 75, No. 68 / Friday, April 9, 2010 / Rules and Regulations.

[20] See, Measure 10. Magnuson–Stevens Fishery Conservation and Management Act Provisions; Fisheries of the Northeastern United States; Northeast (NE) Multispecies Fishery; Amendment 16; Final Rule.

The second consisted of removing several constraints imposed on trade.[21] In particular taxes and caps on quota leasing were removed.[22]

The 2010 Final Report on the Performance of the Northeast Multispecies (Groundfish) Fishery reports the impacts of the New Management Plan in 2010. The report shows a structural change in revenue concentration. After remaining stable from 2007 to 2009, revenue inequality, measured by Gini coefficients, increased by 12 % in 2010. [23] Moreover, the number of active vessels decreased by 32 % (from 658 to 450).[24]

Our main objective is to study the quantitative impact of the quota Leasing and Transfer Program changes approved in Amendment 16 on wealth distribution of active and non active quota holders. Therefore, we calibrate the model so that the equilibrium statistics match the observed quantitative impact on the (observable) revenue distribution and the reduction in active vessels.

4.1 Calibration

We start the calibration by matching the Lorenz curves associated with the Gini coefficients reported in the 2010 Groundfish Final Report.

GBM process and Revenue Lorenz curves The Lorenz curve plots the cumulative proportion of revenues as a function of the cumulative proportion of active vessels. The invariant distribution for the revenue of active vessels, $y(w, r_q, c) = \left(\frac{1 - r_q}{2w}\right) c^{-1}$, generated by the model,[25]

$$F(y) = \int_{y_*}^{y} f(y)dy = 1 - \left(\frac{y_*}{y}\right)^{\xi+1} \left[1 - (1+\xi)\ \ln\left(\frac{y_*}{y}\right)\right],$$

and the cumulative proportion of revenues as a function of the cumulative proportion of the vessels

$$F(c) = \int_{c_*}^{c} f(c)dc = 1 - \left(\frac{c_*}{c}\right)^{\xi-1} \left[(1-\xi)\ \ln\left(\frac{c_*}{c}\right) + 1\right],$$

are functions of the tail index of the of the invariant cost distribution, ξ.

To calibrate the GBM parameters we use the property of the model which states that in the stationary equilibrium the tail index of the cost distribution ξ is a function of μ and σ. Therefore, we calibrate the GBM parameters to match the values of the tail index in 2007 and 2010 that reproduce the Revenue Lorenz curve in each year.[26,27] Figure 2 shows the calibration of Lorenz curves of nominal revenues from groundfish among vessels for 2007

[21] The rationality of this measure is explained as *Removing the cap will facilitate effective use of the leasing program and will provide the ability for some vessels to acquire enough DAS to be profitable. See p 127 of the FINAL Amendment 16 To the Northeast Multispecies Fishery Management Plan. Northeast Multispecies FMP Amendment 16. October 16, 2009*

[22] Those changes are similar to the ones reported by Brandt (2005) in the Atlantic Surf Clam and Ocean Quahog Fishery.

[23] Gini coefficients were 0.663 in 2007; 0.678 in 2008: 0.684 in 2009 and 0.76 in 2010. See Kitts et al. (2011) Table 36, page 63 and Figure 21 page 96.

[24] See Kitts et al. (2011) second paragraph of p. 22.

[25] CDF of revenue are characterized in “Appendix 4”.

[26] “Appendix 7” shows the relationship between Lorenz curves and CDF’s via a simple example.

[27] That is, we want to choose ξ and y_* so that the revenue Lorenz curve generated by our $F(y)$ is the closest to the revenue Lorenz curve generated by the data.

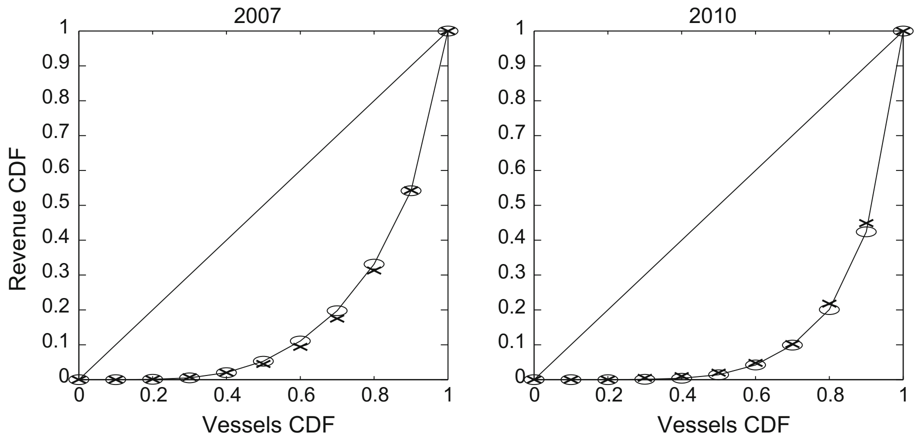

Fig. 2 Lorenz Curve calibration. The Figure shows the calibration of the changes observed in revenue distribution. The *circles* represent the observed data and the *crosses* the prediction by the model. The *curve* shows what percentage (y %) of the total revenues is accounted for by the *bottom* x % of vessels. The percentage of vessels is plotted on the x-axis, and the percentage of revenues on the y-axis. As is well known, the area between the Lorenz curve and the (equal income) straight line is the Gini coefficient. The higher the coefficient, the more unequal the distribution is

Table 1 Revenue distribution (Lorenz curves)

	Bottom 10	20	30	40	50	60	80	90	Top 10
2007									
Data	0.00	0.10	0.30	1.00	2.80	5.70	10.20	22.30	57.60
Model	0.00	0.50	0.50	1.00	2.80	5.40	11.60	23.10	55.10
2010									
Data	0.10	0.40	1.50	3.29	5.79	8.68	13.37	21.06	45.81
Model	0.10	0.60	1.30	2.50	4.90	8.10	13.80	23.00	45.70

and 2010. The circles represent the observed data and the crosses the prediction by the model. As is well known, the area between the Lorenz curve and the (equal income) straight line is the Gini coefficient.[28]

We were unable to find data to compute the vessel lifespan for the Northeast Multispecies (Groundfish) Fishery, so we calibrate the vessel lifespan to match the value of the drift estimated by Weninger and Just (2002) in 2007.[29] The other GBM parameters that match the Lorenz curve can be computed using propositions 2 and 3. That is, the process μ and σ is chosen by minimizing the distance of the Lorenz curve generated by the model to the one generated by the data. Table 1 summarizes how the model matches the Lorenz curves and Table 2 summarizes targets and GBM parameters.

[28] Note that a Lorenz curve always starts at (0,0) and ends at (1,1), independently of the support of the variable.

[29] Weninger and Just (2002) assumed that c is the unit operating cost. This unit operating cost is assumed to be distributed over a bounded support $[\underline{c}, c_*)$. Therefore, the GBM process (μ and σ) and the support, $\underline{c}$, can be estimated using average variable cost data. They use a sample of 22 vessels from the Mid-Atlantic surf clam and ocean quahog fishery to estimate $\mu = 0.04$, $\sigma^2 = 0.16$ and $c_*0 = .62$.

Table 2 GBM parameters calibration

Parameter			Target
Stochastic process in 2010			
μ_{2010}	Drift	0.0400	Weninger and Just (2002)
μ_{2007}	Drift	0.0431	Lorenz$_{2007}$
σ^2	Volatility	0.0121	Lorenz$_{2010}$

Table 3 Other model parameter calibration

Discount factor, ($\rho = 0.05$)					
Parameter			Target		
Costs					
c_f	Fix cost	0.31	Δfleet$_{2010}$	$1 - M$	0.32
c_e	Entry cost	9.19	Margin with entry	$(p - r_q)$	0.60

The size of the revenue support in our data (computed as top 10 / bottom 10) is $\simeq$ 576 and 458 % in each year. Although this support is higher that the revenue support computed by Weninger and Just (2002), we estimate that σ^2 is 1.21 %, i.e. lower than the 16.00 % estimated by Weninger and Just (2002).[30] That is, higher vessel samples reduce volatilities.[31] Finally, the model shows that the reform reduces drift ($\mu_{2010} < \mu_{2007}$).[32]

It is important to notice that the reduction in the drift parameter (from 0.0431 to 0.0400) could be interpreted as an increase in efficiency due to the removal of restrictions on leasing quotas. This increase in efficiency can be rationalized, at least partially, as the result of having more complete markets that foster investment in more efficient production processes.

Other parameters Given the GBM process, it is necessary to calibrate three parameters c_f, c_e and ρ.[33] We start by selecting a value of the annual interest rate $\rho = 0.04$ which is standard for the US economy in macroeconomic literature.[34] We calibrate the other two parameters by solving the equilibrium of the model and making sure the equilibrium statistics match statistics from the fishery. In particular, we calibrate the entry cost c_e and the fixed cost to match the reduction observed in the number of active vessels between 2007 and 2010, and the leasing quota prices in 2010.[35] Finally, the TAC is normalized to 1 so that in each year $q = 1/M$. The parameter values are summarized in Table 3.

Table 4 shows the equilibria generated with the parameters. The model reduces the abandonment threshold by 31 % to match the fleet squeeze. The average cost, $\overline{c} = \int cg(c)dc$, is reduced by 50 %. In our model c^{-1} is a TFP shifter that can not be estimated directly from the data. But there is indirect evidence of these efficiency gains. For example, average trip costs

30 The support of the distribution is given by the size of c_*, which is endogenous in the model. They find that the abandonment cost threshold is $c_* = 5.57$, 278 % greater than the cost of the most efficient vessel. They set the price at 7.60. Therefore, the size of the revenue, $p - c$, support is equal to $(7.60 - 0.62)/(7.60 - 5.57) \simeq 344$ %.

31 The 2007 and 2010 Lorenz curves were obtained using data from 658 and 450 vessels, respectively.

32 This is consistent with the empirical evidence. For example Morrison Paul et al. (2009) found significant growth in economic productivity after a property rights-based management reform.

33 The formulas used for c_f and c_e are given in “Appendix 5”, and the formulas used to compute the model equilibrium are given in “Appendix 6”.

34 See, for instance Restuccia and Rogerson (2008).

35 See Kitts et al. (2011) Table 26, page 55.

Table 4 Model equilibria

		2007	2010
		Efficiency gains	
$\pi(w, r_q)$	Profit (per unit of productivity)	0.5556	0.6414
c_*	Maximum cost	1.2000	0.8247
$\bar{c}$	Average cost	0.9037	0.4478
		Fleet squeeze	
M	Industry size	1.0000	0.6839
q	Effective quota utilized by active firms	1.0000	1.4622
		Endogenous prices	
w	Effort cost	0.2988	0.4117
p_q	Permanent sell price	–	10.0000

per day for vessels between 50' and <75' fell by 26 % between 2007 and 2010 and TFP measures for vessels affected by the regulatory changes were up by 12 %.[36] Moreover, observe that active firms are more productive and demand more than one permit (as $q = 1.4622$). Finally, the equilibrium quota price is 10.00.

5 Wealth Distribution

We quantify the impact of allowing participation in the Leasing and Transfer Programs without the requirement of owning an active vessel on firm value and wealth distribution by comparing these statistics for 2010, (when quotas were attached to vessel owners) with the equivalent statistics for 2007 (when quotas were attached to vessels).

In both economies the value of the firms, $W(c)$, includes the value of the vessel (the capital) and the value of the quota.[37] In both economies some less productive *active* firms (vessels) lease part of the quota, i.e. $q - y(c) > 0$.

The key differences between these economies are related to the number of assets in each one. When the property right is attached to the vessel there is only one asset in the economy: the vessel with the attached permit to operate in the industry. Firms operate capital and stay *active* if they find it optimal to pay the idling cost, c_f. Note that the marginal firm (the less efficient vessel) is indifferent between paying the idling cost to fish the minimum amount to be considered active, $y(c_*)$, which enables it to lease the remaining quota, $q - y(c_*)$, and exiting the market. This marginal firm makes negative instantaneous profits and the total expected value of operating the vessel is zero. Therefore, if the marginal active firm decides to leave the market, it obtains this value, $W(c_*) = 0$.

Figure 3 shows that allowing total transferability of quotas shifts the value function upwards via the exit constraint $W(c) \geq p_q q$. It also shows that increasing the value of exiting changes the incentives to exit.

When the property right is attached to the owner of the vessel and divorced from ownership of an active vessel it can be traded as a separate asset. Active firms can decide to *cease activities* and become a lessors of quotas if they find it optimal to (permanently) lease their quotas. The

[36] See Kitts et al. (2011) Table 15, page 47 and Table 20, page 51.

[37] Note that, as in Weninger and Just (2002), firms are operating one unit of capital.

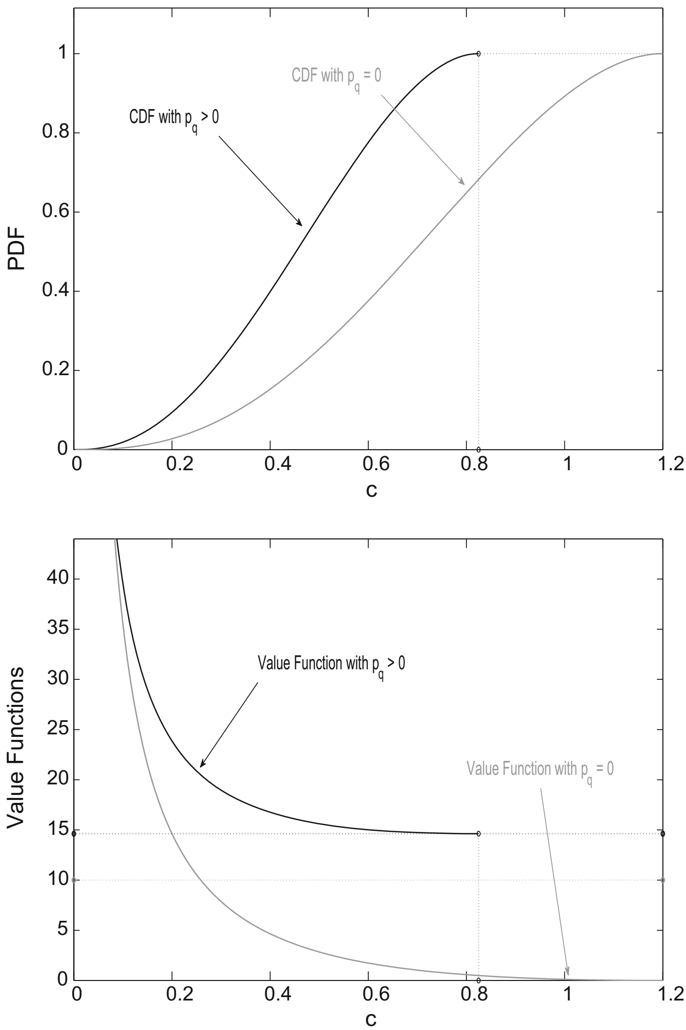

Fig. 3 Changes in cost distribution and value functions under the two market structures. A decrease in maximum cost (a fleet squeeze) is shown that generates an increase in productivity (the efficiency gain)

marginal firm (the least efficient vessel) is indifferent between paying the idling cost to fish the minimum amount required for it to be considered active $y(c_*)$, which allows it to lease the remaining quota, $q - y(c_*)$, and permanently leasing its quota without paying the idling cost. Note that when firms are allowed to lease quotas without being active the marginal firm becomes more efficient.[38] Moreover, by allowing active vessels to trade quotas as a separate asset the value of the active firms is increased. If the marginal active firm decides to cease its activity it obtains the value of selling the right, $W(c_*) = p_q q$, without paying the idle cost. Figure 3 shows the impact of changes in efficiency (in the abandonment threshold) on cost distribution and on the value functions.

[38] It is clear that a firm which is fishing the "minimum" to be considered active $y(c_*) \to 0$, finds it optimal to lease its quota without paying the idle cost, rather than paying the idling cost to lease the quota.

Table 5 Wealth distribution with fully tradable ITQs (active firms)

	Bottom 5 %	Quartiles				Top 5 %	Mean	Gini
		q1	q2	q3	q4			
2007								
Wealth (%)	0.01	0.04	0.85	5.46	93.65	67.66	100	0.88
Mean	0.01	0.02	0.03	0.22	3.75	13.53	1.00	
2010								
Wealth (%)	0.73	3.92	11.47	20.73	63.87	30.82	100	0.54
Mean	0.14	0.16	0.46	0.83	2.55	6.16	1.00	

We are interested in exploring the distributional impact of these changes. We divide the section into three parts. First, we study the impact on the value of active firms. More precisely, we show that by raising the value at the margin, the change in transferability generates a redistributive effect which is similar to giving a fixed transfer to each firm. Second, we construct the wealth distribution for the whole economy: including the inactive *quota owners* and the active firms; and third, we compare the impact of (potential) changes in innovation rates on wealth distribution.

5.1 The Effect of a Lump Sum Transfer on Active Firms

Table 5 represents the wealth distribution of active firms in 2007 and 2010. For instance, the numbers under "bottom 5 %" represent the proportion of the total wealth that goes to the poorest 5 % of the active firms and the numbers under the quartiles represent the amount of wealth that goes to the corresponding quartile.[39]

In 2007 the model generates more inequality in wealth than in income. That is, the Gini coefficient of the wealth distribution (0.88) is higher than the Gini coefficient of income distribution (0.70).[40] This is a stylized fact of the US Economy.[41] The top 5 % in the mean wealth ratio is 13.53.[42] Note also that firms at the bottom are very poor. This is because they are obtaining negative profits and waiting for better times.[43]

Table 5 shows that wealth distribution in 2010 is less concentrated. This reduction in inequality comes from the exit condition, which implies $W(c) \geq p_q q$, which in turn implies that with permanent transferable quotas the marginal firm has a positive value. That is, giving full transferability to fishing rights is equivalent to giving a lump sum transfer (of the same amount) to all firms in the market independently of their wealth levels. This reduces inequality as the hypothetical transfer to the poorer firms is larger in proportion to their original wealth than that given to the richer ones.

Consider an example to explain the impact of a lump sum transfer on the Gini coefficient and the Lorenz curve. The example shows that this redistributive mechanism reduces the Gini coefficient. Consider an economy with three agents and an initial endowment of $W_0 = [0 \quad 5 \quad 10]$. That is, agent 1 has zero wealth, agent 2 has five units and agent 3 has ten

39 We compute the wealth distribution by using $W(c)$ and $f(c)$ for each economy.

40 "Appendix 7" describes the Brown Formula used to compute the Gini coefficient.

41 See Diaz-Gimenez et al. (2011).

42 This ratio for the total US Economy (i.e. including households) is 8.1631

43 This is a well known result. See Weninger and Just (2002).

Table 6 Gini Index: impact of a transfer

	Terciles			Mean	Gini
	t1	t2	t3		
Initial wealth (%)	0.00	0.33	0.66	100	0.44
Wealth after a transfer (%)	0.17	0.33	0.50	100	0.22

Table 7 Wealth levels

	Inactive landlords	Active vessels						
		Bottom 5 %	Quartiles				Top 5 %	Gini
			q1	q2	q3	q4		
Mass with $p_q = 0$	31.64	3.42	17.09	17.09	17.09	17.09	3.42	
Wealth $p_q > 0$	10.00	14.62	15.76	46.05	83.23	256.44	618.75	0.48

units of wealth. If each agent receives a transfer of five units of wealth, the new endowment distribution is $W_1 = [0 \;\; 5 \;\; 10]$. Table 6 shows that the Gini coefficient is 0.44 before the transfer and 0.22 after the transfer. Moreover, if the Lorenz curves of these two economies are plotted a transfer of these characteristics shifts the Lorenz curve to the right.

5.2 Wealth Distribution for Quota Owners

Table 5 compares only the wealth levels of active firms. However, in the 2010 economy there is a new class of quota landlords who decide to cease activity but lease their quotas. Table 7 compares total wealth changes.[44]

The model predicts that firms that cease activities (31.64 % in Table 7) are the least efficient in the 2007 economy. In the 2010 economy, they find it optimal to cease activities and permanently lease their ITQs at a price p_q (10.00). The wealth of these small owners is therefore multiplied by 500. In the same table, the top 5 % represents the most efficient, which own many quotas. Their wealth is therefore multiplied by a factor of less than 10. Therefore, total wealth is less concentrated than the wealth of active vessels.

5.3 Wealth and Innovation Rate

The experiments described below explore the impact of higher innovation rates on the wealth inequality calculated by our model.[45] We expect that better functioning markets, with more complete and transferable property rights, foster innovation. Table 8 reports Gini coefficients for active and total owners, and the percentage of exiting firms for four different levels of μ. Notice that if more tradability of ITQs generates an increase in the innovation rate this is a force for increasing the level of inequality. In our model, in order to generate more inequality than in the case with restricted tradability (remember that in that case the Gini coefficient

[44] We computed wealth distribution in 2010 by using

$$W = \begin{cases} p_q q & \text{if } c \in (c_*^{2010}, c_*^{2007}] \\ W^{2010}(c) & \text{if } c \in (0, c_*^{2007}] \end{cases} .$$

[45] We are indebted to an anonymous referee for this suggestion.

Table 8 Wealth and innovation rate

Innovation rate	Active vessels	Owners	
	Gini	Exit	Gini
μ	0.5414	31.64	0.4853
$3/4\mu$	0.6613	52.94	0.6576
$2/3\mu$	0.7093	66.01	0.7547
$1/2\mu$	0.8016	98.36	0.9748

was 0.88), the innovation rate must increase by 50 % (compared to the innovation rate of just 6 % suggested by the data), which would be associated with a fleet shrinkage of around 90 %.

6 Conclusions

Much of the reluctance to use individual transferable quotas in the US is due to the concern that ITQs will change participants' relative positions in the fishery and, in particular, to the fear that small-scale fishermen will be disadvantaged relative to larger producers. However Brandt (2005) shows that in the US mid-Atlantic clam fishery no segment of the industry was disproportionately adversely affected by the regulatory change. In this paper we build a formal model that supports these findings. Moreover, we find that allowing fully transferable rights is equivalent to giving a lump sum transfer (of the same amount) to all firms in the market which is independent of their wealth level. This reduces inequality as the transfer to the poorer is larger in proportion to their original wealth than that to the richer.

In our model heterogeneity is generated by firm-specific shocks to production opportunities. However, the same result could be achieved with other firm-specific shocks, e.g. differences in prices and demands driven by the composition of catches and/or quality.[46] In that case, perhaps a more precise statement of the results would be that if agent heterogeneity is high enough then trading of permits does not necessarily increase wealth inequality.

Finally, as in Weninger and Just (2002) we introduce capital as the static decision as to whether to buy a vessel (interpreted as consisting of a unit of capital). Given that our model is concerned with the stationary equilibrium we can abstract from capital dynamics.[47] However, capital dynamics are important for understanding transitions.[48] We leave this analysis for future research.

Appendix 1: Proof of Proposition 1

Let $J(x, t)$ be the Value function associated with the following problem

$$J(c, t) = \max_{d \in \{stay, exit\}} \left\{ \max_{u} \int_{t}^{t+dt} \pi(c, u) e^{-\rho s} ds + E_{dc} J(c + dc, t + dt), \, p_q q \right\},$$

[46] For instance, in Da-Rocha and Pujolas (2011a) heterogeneity comes by differences in the species composition of vessel catches.

[47] Veracierto (2001) founds that capital is not important for understanding the stationary equilibrium. For this reason, the literature refrains from considering capital, or introduces it as a static decision.

[48] See Lai (2007) and Hannesson (1996).

where c follows a Geometric Brownian Motion, $dc = \mu c dt + \sigma c dw$, u is a vector of control variables and $p_q q$ is the termination payoff. Using a Taylor expansion, the following can be written

$$\begin{aligned} J(c+dc, t+dt) &= J(c,t) + J_t(c,t)dt + J_c(c,t)dc \\ &\quad + \frac{1}{2}\left\{J_{tt}(c,t)dt^2 + J_{tc}(c,t)dtdc + J_{cc}(c,t)dc^2\right\}. \end{aligned}$$

Using Ito's calculus and taking limits, $dt \to 0$, the *Hamilton–Jacobi–Bellman* equation is obtained

$$-J_t(c,t) = \max_{d \in \{stay, exit\}} \left\{ \max_u \pi(c,u)e^{-\rho t} + \mu J_c(c,t) + \frac{\sigma^2}{2} J_{cc}(c,t),\, p_q q \right\}.$$

Given that $\pi(c,u)$ is autonomous, there is a stationary solution $J(c,t) = e^{-\rho t} W(c)$. Then the (stationary) Hamilton–Jacobi–Bellman equation is

$$\rho W(c) = \max_{d \in \{stay, exit\}} \left\{ \max_u \pi(c,u) + \mu c W'(c) + \frac{\sigma^2 c^2}{2} W''(c),\, p_q q \right\}.$$

Assume that c_* is the optimal exit point such that

$$d = \begin{cases} \text{stay} & \text{if } c \leq c_* \\ \text{exit} & \text{if } c > c_*, \end{cases}$$

and $\max_u \pi(c,u) = \pi(w, r_q)c^{-1}$. Then the optimal policy satisfies

$$\rho W(c) = \pi(w, r_q)c^{-1} + (r_q q - c_f) + \mu c W'(c) + \frac{\sigma^2 c^2}{2} W''(c),$$

subject to the boundary conditions $W(c_*) = p_q q$ and $W'(c_*) = 0$. Guessing that $W(c) = A_1 c^\beta + A_2 c^{-1} + A_3$, the JHB equation becomes equal to:

$$\begin{aligned} \rho\left(A_1 c^\beta + A_2 c^{-1} + A_3\right) &= \pi(w, r_q)c^{-1} + (r_q q - c_f) \\ &\quad + \mu c\left(\beta A_1 c^{\beta-1} - A_2 c^{-2}\right) \\ &\quad + \frac{(\sigma c)^2}{2}\left(\beta(\beta-1)A_1 c^{\beta-2} + 2A_2 c^{-3}\right). \end{aligned}$$

Rearranging terms we have

$$\begin{aligned} 0 &= \beta^2 - \left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)\beta + \frac{2\rho}{\sigma^2}, \\ A_2 &= \frac{\pi(w, r_q)}{\rho + \mu - \sigma^2}, \\ A_3 &= (r_q q - c_f). \end{aligned}$$

Finally we use the boundary conditions

$$\begin{aligned} W(c_*) &= A_1 c^\beta + \frac{\pi(w, r_q)c^{-1}}{\rho + \mu - \sigma^2} + (r_q q - c_f)\bigg|_{c=c_*} = p_q q, \\ c_* W'(c_*) &= \beta A_1 c^\beta + \frac{\pi(w, r_q)c^{-1}}{\rho + \mu - \sigma^2}\bigg|_{c=c_*} = 0, \end{aligned}$$

to obtain A_1 and c_*. That is

$$c_* = \frac{(1+\beta)}{\beta} \frac{\rho}{(\rho+\mu-\sigma^2)} \left(\frac{\pi(w, r_q)}{\rho p_q q + c_f - r_q q} \right),$$

$$A_1 = \left(p_q q - \frac{(r_q q - c_f)}{\rho} \right) \frac{\beta}{1+\beta} \left(\frac{1}{c_*} \right)^{\beta}.$$

Hence, the value function of an individual is

$$W(c) = \left(p_q q - \frac{(r_q q - c_f)}{\rho} \right) \frac{\beta}{1+\beta} \left(\frac{c}{c_*} \right)^{\beta} + \frac{\pi(w, r_q) c^{-1}}{\rho+\mu-\sigma^2} - \left(\frac{c_f - r_q q}{\rho} \right).$$

Appendix 2: Proof of Proposition 2

Applying Laplace transforms in $f''(x) - \gamma_1 f'(x) + \gamma_2 f(x) = 0$, gives:

$$(s^2 - \gamma_1 s + \gamma_2)\mathscr{L}[f(x)] - (s - \gamma_1) f(0) - f'(0) = 0.$$

Using the boundary condition $f(0) = 0$ we find:

$$\mathscr{L}[f(x)] = \frac{f'(0)}{(s^2 - \gamma_1 s + \gamma_2)}.$$

Note that $\gamma_1 = \frac{2\hat{\mu}}{\sigma^2} > 0$ and $\gamma_2 = \frac{2\varepsilon}{\sigma^2} > 0$ implies that only solutions with positive roots can exist. The solution depends on the number of (positive) roots of the equation $s^2 - \gamma_1 s + \gamma_2 = 0$. It is clear that from this equation, the solution must satisfy

$$r_i = \frac{\gamma_1 \pm \sqrt{\gamma_1^2 - 4\gamma_2}}{2} \quad \forall i = 1, 2. \tag{1}$$

There are then two possible solutions. One implies $r_1 \neq r_2 = r$ and the other $r_1 = r_2 = r$. We prove that the first possibility cannot be a solution of our problem by contradiction. Consider a solution with two different roots, so that the discriminant $\gamma_1^2 - 4\gamma_2$ does not vanish, implying that $\gamma_1^2 \neq 4\gamma_2$. With two different (positive) roots the solution of the second order differential equation becomes:

$$\mathscr{L}[f(x)] = \frac{f'(0)}{(s - r_1)(s - r_2)}.$$

We obtain the solution by solving the Laplace inverses given by:

$$f(x) = \mathscr{L}^{-1}\left[\frac{f'(0)}{(s - r_1)(s - r_2)} \right] = \frac{f'(0)}{(r_1 - r_2)} \left(e^{r_1 x} - e^{r_2 x} \right).$$

Note that $f'(0) = r_1 r_2 \neq 0$, and this implies a contradiction.

Therefore, the solution satisfies $r = r_1 = r_2$. This implies that $\gamma_1^2 - 4\gamma_2 = 0$, and that $r_i = \frac{\gamma_1}{2} \quad \forall i = 1, 2$. This solution gives us

$$f(x) = \mathscr{L}^{-1}\left[\frac{f'(0)}{(s - r)^2} \right] = f'(0) x e^{rx}.$$

Appendix 3: Proof of Proposition 4

First note that $\int_0^{c_*} c^a g(c)dc = \left(\frac{1+\xi}{1+\xi+a}\right)^2 c_*^a$. Taking expectations, and using the value of c_*, we have that

$$\begin{aligned}
W^e &= \int_0^{c_*} W(c)g(c)dc - wc_e - p_q q \\
&= \frac{(\xi+1)^2}{(1+\beta)(\xi+1+\beta)^2}\left(p_q q - \frac{(r_q q - c_f)}{\rho}\right) + \frac{(\xi+1)^2}{\xi^2}\left(\frac{(p-r_q)^2}{4w}\frac{c_*^{-1}}{\rho+\mu-\sigma^2}\right) \\
&\quad + \frac{(r_q q - c_f)}{\rho} - wc_e - p_q q \\
&= \left(p_q q - \frac{(r_q q - c_f)}{\rho}\right)\left[\frac{(1+\xi)^2}{(1+\beta+\xi^2)(1+\beta)} + \frac{\beta(1+\xi)^2}{(1+\beta)\xi^2} - 1\right] - wc_e.
\end{aligned}$$

Then, from the f.o.c. of the entering firm's problem we have

$$\left(p_q - \frac{r_q}{\rho}\right)\left[\frac{(1+\xi)^2}{(1+\beta+\xi^2)(1+\beta)} + \frac{\beta(1+\xi)^2}{(1+\beta)\xi^2} - 1\right] = 0 \Rightarrow p_q = \frac{r_q}{\rho}.$$

Appendix 4: Cumulative Distribution Function

Revenue $y(w, r_q, c) = \left(\frac{1-r_q}{2w}\right)c^{-1}$ is non linear in c. However, the invariant distribution of revenue is a simple change in the power of the invariant cost distribution. That is,

$$f(y) = -\frac{(\alpha-1)^2}{y_*}\left(\frac{y_*}{y}\right)^{\xi+2}\ln(y_*/y).$$

We calculate

$$F(y) = \int_{y_*}^{y} f(y)dy = \int_{y_*}^{y} -\frac{(\alpha-1)^2}{y_*}\left(\frac{y_*}{y}\right)^{\xi+2}\ln(y_*/y)dy.$$

Trivial manipulation implies that the cumulative distribution function is

$$F(y) = \int_{y_*}^{y} f(y)dy = 1 - \left(\frac{y_*}{y}\right)^{\xi+1}\left[(-\xi-1)\ln\left(\frac{y_*}{y}\right) + 1\right].$$

Appendix 5: Calibration

We proceed as follows. Given μ and σ, first we set $M = 1 - \Delta\text{fleet}_{2010}$ and $p - r_q = \text{margin}_{2010}$, and we compute

$$\begin{aligned}
c_f &= \frac{\text{margin}_{2010}}{2(1-\Delta\text{fleet}_{2010})^2}\left(\frac{\xi}{\xi+1}\right)^2\frac{(1+\beta)}{\beta}\left(\frac{\rho}{(\rho+\mu-\sigma^2)}\right),\\
c_e &= \frac{1}{\text{entry}}\left[\frac{1}{(1-\Delta\text{fleet}_{2010})} - 2c_f\left(\frac{\xi+1}{\xi}\right)^2\frac{\beta}{(1+\beta)}\left(\frac{(\rho+\mu-\sigma^2)}{\rho}\right)\right],\\
w &= \frac{c_f}{\rho c_e}\left[\frac{(1+\xi)^2}{(1+\beta+\xi)^2(1+\beta)} + \frac{\beta(1+\xi)^2}{(1+\beta)\xi^2} - 1\right],\\
c_* &= \frac{(\text{margin}_{2010})^2}{4w}\frac{1}{c_f}\frac{(1+\beta)}{\beta}\left(\frac{\rho}{(\rho+\mu-\sigma^2)}\right).
\end{aligned}$$

Finally we compute c_* in 2007 to match $\Delta\text{fleet}_{2010}$. That is

$$\Delta\text{fleet}_{2010} = \int_{c_*}^{c_*^{2007}} -\frac{(1+\xi_{2007})^2}{c_*^{2007}}\log(x/c_*^{2007})\left(\frac{x}{c_*^{2007}}\right)^{\xi_{2007}} dx.$$

Appendix 6: Solving for the Equilibrium

Given μ and σ, the equilibrium, w, r_q p_q, M, and c_*, are given by the following set of five equations. First, entry condition

$$w = \frac{1}{c_e}\left(\frac{c_f}{\rho}\right)\left[\frac{(1+\xi)^2}{(1+\beta+\xi)^2(1+\beta)} + \frac{\beta(1+\xi)^2}{(1+\beta)\xi^2} - 1\right].$$

From the labour market condition, we can obtain the mass of firms M,

$$1 - M\varepsilon \times c_e = M\int_0^{c_*} l(c)g(c)dc = M\left(\frac{(p-r_q)}{2w}\right)^2\left(\frac{\xi+1}{\xi}\right)^2 c_*^{-1}.$$

From the output market, we have

$$1 = M\overline{q} = M^2\int_0^{c_*} y(c)g(c)dc = M^2\frac{(p-r_q)}{2w}\left(\frac{\xi+1}{\xi}\right)^2 c_*^{-1}.$$

and the maximum cost c_*, is

$$c_* = \frac{(1+\beta)}{\beta}\frac{(p-r_q)^2}{4w}\frac{1}{(\rho+\mu-\sigma^2)}\left(\frac{\rho}{c_f}\right),$$

and p_q is such that $p_q = \dfrac{r_q}{\rho}$. Simple manipulation allows us to find the close-form solution:

$$w = \frac{c_f}{\rho c_e}\left[\frac{(1+\xi)^2}{(1+\beta+\xi)^2(1+\beta)} + \frac{\beta(1+\xi)^2}{(1+\beta)\xi^2} - 1\right],$$

$$\frac{1}{M} = c_e\varepsilon + 2c_f\left(\frac{\xi+1}{\xi}\right)^2 \frac{\beta}{(1+\beta)}\left(\frac{(\rho+\mu-\sigma^2)}{\rho}\right),$$

$$(p - r_q) = 2c_f M^2\left(\frac{\xi+1}{\xi}\right)^2 \frac{\beta}{(1+\beta)}\left(\frac{(\rho+\mu-\sigma^2)}{\rho}\right),$$

$$c_* = \frac{(p-r_q)^2}{4c_f w}\frac{(1+\beta)}{\beta}\left(\frac{\rho}{(\rho+\mu-\sigma^2)}\right).$$

Appendix 7: The Computation of Gini Coefficients and Lorenz Curve

In order to compute the Gini coefficients in our calibrations we use the approximation by trapezoids known as Brown's formula. Formally, define $p(n)$ as the density and $P(n)$ as the accumulated proportion of the population variable, for$n = 0$, with N being the types of individuals differentiated by wealth (and ordered from least to greatest wealth), with $P(0) = 0$ and $P(N) = 1$. Define as $w = 0 \ldots W$ the different wealth levels (where wealth is ordered in a non decreasing fashion) and let $f(w)$ be the density and $F(w)$ be the cumulative proportion of the wealth variable. The Gini coefficient can then be defined as

$$Gini = 1 - \sum_i (P(i) - P(i-1))(F(i) + F(i-1))$$

An application that measures the effect of a lump sum transfer on the Gini coefficient is presented in Table 9. Column 1 is the amount transferred. Column 2 is the proportion of the population in each wealth level. Columns 3 and 4 are the wealth levels before and after the transfer, respectively. Column 5 represents the cumulaivte distribution of people and columns 6 and 7 the cumulative distribution of wealth before and after the transfer. The rest of the columns are helpful in computing Brown's formula. It is immediately apparent by straightforward application of the formula that the Gini coefficient is 0.44 before the transfer and 0.22 after it.

The Lorenz curve plots the cumulative proportion of wealth as a function of the cumulative proportion of the population. Table 10 shows the calculation and the effect on the Lorenz curve of the transfer discussed in Table 9. As before, Column 1 is the amount transferred. Column 2 is the proportion of the population in each wealth level. Columns 3 and 4 are the wealth levels before and after the transfer, respectively. Column 5 and 6 represent the

Table 9 Gini Index: impact of a transfer

Transfer	p(n)	w0	w1	P(i)	F(i)	F(w1)	A = P(i) − P(i−1)	B = F(i) + F(i−1)	
5.00	0.33	0.00	5.00	0.33	0.00	0.17	0.33	0.00	0.17
5.00	0.33	5.00	10.00	0.67	0.33	0.50	0.33	0.33	0.67
5.00	0.33	10.00	15.00	1.00	1.00	1	0.33	1.33	1.50
Total	1.00	15.00	30.00			1	–	–	–

Table 10 Lorenz curve: impact of a transfer

Transfer	p(n)	w0	w1	f(i)	f(i)	p(i) + p(i−1)	f(i) + f(i−1)	
5.00	0.33	0.00	5.00	0.00	0.17	0.33	0.00	0.17
5.00	0.33	5.00	10.00	0.33	0.33	0.66	0.33	0.67
5.00	0.33	10.00	15.00	0.67	0.50	1	1	1
Total	1.00	15.00	30.00	1.00	1.00	–	–	–

proportion of wealth belonging to each type of agent before and after the transfer. The Lorenz curve corresponding to the case without the subsidy plots column 7 in the horizontal axis and column 8 in the vertical axis (the Lorenz curve corresponding to the economy with the subsidy is symmetrically defined using column 9).

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